U.G. Sem III

31. 10. 2025

Bernoulli's equation / Reducible Linear equation

Such equations can be converted to Linear equation of the form $\frac{dy}{dx} + Py = Q$ by some suitable substitution.

1. $$\frac{dy}{dx} + \frac{1}{3}y = \frac{1}{3}(1-2x)y^4$$

Dividing both sides by y^4, we have

$$\Rightarrow y^{-4}\frac{dy}{dx} + \frac{1}{3}y^{-3} = \frac{1}{3}(1-2x) \quad \text{——(1)}$$

Put $y^{-3} = z$ —(2) Diff. w.r.to x, we get

$$-3y^{-4}\frac{dy}{dx} = \frac{dz}{dx} \Rightarrow y^{-4}\frac{dy}{dx} = -\frac{1}{3}\frac{dz}{dx} \quad \text{—(3)}$$

using eq(2) and eq(3) in eq(1), we get

$$\Rightarrow -\frac{1}{3}\frac{dz}{dx} + \frac{z}{3} = \frac{1}{3}(1-2x)$$

$$\Rightarrow \frac{dz}{dx} - z = 2x-1$$ which is in the form of a linear equation.

Here, $P = -1$, $Q = 2x-1$

31.10.25
(contd.)

$\therefore$ I.F. $= e^{\int P\,dx} = e^{-\int dx} = e^{-x}$

Hence the soln. is given by

$$z \times I.F = \int Q \times I.F.\, dx$$

$$\Rightarrow z \times e^{-x} = \int (2x-1)\, e^{-x}\, dx$$

$$\Rightarrow z \times e^{-x} = \int 2x\, e^{-x}\, dx - \int e^{-x}\, dx$$

$$\Rightarrow z e^{-x} = 2\left[x\int e^{-x}\, dx - \int\left[\frac{d(x)}{dx}\int e^{-x}\, dx\right]dx\right] + e^{-x} + K$$

$$= 2\left[-x e^{-x} - e^{-x}\right] + e^{-x} + K$$

$$\Rightarrow z e^{-x} = -2x e^{-x} - e^{-x} + K$$

$$\Rightarrow z = -2x - 1 + K e^{x}$$ But $z = y^{-3}$ from (2)

$$\Rightarrow \frac{1}{y^3} = -(2x+1) + K e^{x}$$

$$\Rightarrow 2x + 1 + \frac{1}{y^3} = K e^{x}$$ This is the required soln.